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Practical slip circle method of slices for calculation of bearing capacity factors

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Abstract

The slip circle method of slices is commonly used in the analyses of slope stability and bearing capacity for multi-layered ground. However, in the case of ground consisting of horizontal sandy layer, it is known that modified Fellenius' method tends to underestimate the factor of safety, while simplified Bishop's method tends to overestimate the factor of safety. In this study, a new slip circle method was proposed for the purpose of improving the accuracy of the analysis for a ground consisting of sand and clay layers. In the proposed method, β of the ratio of inter-slice shear force to inter-slice normal force i.e $\tan(\beta\alpha_i)$ is assumed constant as 0.25 for all slices. This is named as circle bearing capacity factor (CBCF) method. It was found that the bearing capacity factors, N_c, N_q, and N_γ calculated for shallow foundation on horizontal ground from CBCF method agreed well with that obtained from the plastic solution. The back-analyses carried out for a few case studies on the stability of slopes on earth structures found in sand and clay layers showed that the factor of safety calculated from CBCF method explains the actual performance of earth structures well. The proposed CBCF method proves it reliability in calculating bearing capacity for shallow foundations. This was achieved from the results obtained from centrifugal model test, which were carried out for dense sand layer overlying soft clay with various conditions by Okamura et al. (1998). It was examined that the factor of safety calculated for the stability of slopes from CBCF method can explain the actual performance of geotechnical structures constructed on ground consisting of sand and clay layers.

Key words: Stability analysis; Slip circle; Bearing capacity; Soft ground

1. Introduction

Over the years, the slip circle method of slices is known to be the most popular among the designers and researches in the design works of earth structures. In this method, a potential failure mass is divided into number of finite vertical slices and the equilibrium of each slice is considered in determination of the factor of safety (Taylor, 1948; Tshebotarioff, 1951; Bishop, 1955). Fig. 1 illustrates a trial failure mass divided into number of slices and a slice with the unknown forces acting on it, including the resultants V_i and E_i' of shear and normal effective forces along sides of the slice, as well as the resultants T_i and N_i' of shear and normal effective forces, respectively. As the slip circle method of slices can be applied for various shapes or non-homogeneous

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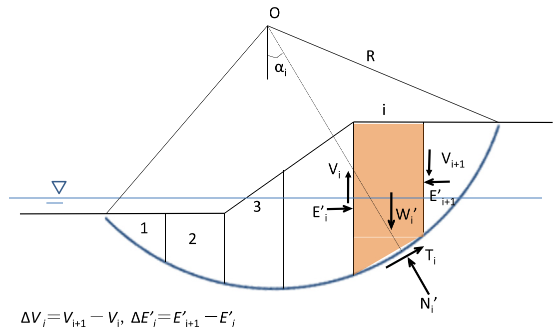

Fig. 1. Forces acting on a slice in the slip circle method.

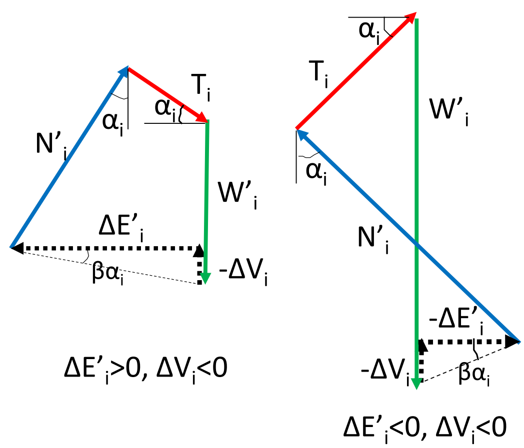

Fig. 2. Equilibrium of forces acting on a slice in the proposed method.

ground, this method has been used extensively in geotechnical engineering practice since its development, although more sophisticated methods such as a finite element method (Matsui and San, 1992; Noda et al., 2007) are available in the literature.

A number of methods are currently being applied to numerous slope stability problems based on the slip circle method of slices. However, each method is different from another and based on different assumptions on the forces acting upon the sides of the slices. In the simplest method of slices (known as Fellenius' method or Swedish method of slices), the resultant of all inter-slice forces is assumed to be consistent with the direction of failure arc for the slice. With this assumption, there are no resultant forces acting on the sides of a slice in the direction normal to the failure arc. Therefore, the factor of safety can be calculated without considering the forces acting on the sides of slices. In the modified Fellenius' method (M.F. method), the effective weight of slice is used for the slices where the potential slip surface is below the water table instead of considering the resultant of static pore water pressure along the failure arc (Nakase, 1967; Ugai and Hosobori, 1985). In the simplified Bishop's method (S.B. method), it is assumed that the forces acting on the sides of each slice have zero resultant in the vertical direction (Bishop, 1955). This method also used in the geotechnical practices by many designers and researchers over the years. Cheng et al., 2010 carried out extensive studies on inter-slice force function $f(x)$, the function of inter-slice normal and shear forces of the slices, and emphasis the importance of it in determining the factor of safety.

M.F. method has been used as a conventional method in design of geotechnical structures constructed on soft ground. In 1960s, Nakase carried out an extensive study on this method and successfully applied to large-scale slides of soft ground in the coastal areas (Nakase, 1967). However, when the ground consists of only sand or gravel or consists of upper sand layer and lower clay layer, it is known that M.F. method underestimates the factor of safety, while, S.B. method often overestimates the factor of safety (Turnbull and Hvorslev, 1967; Yamaguchi, 1984).

Similar shortcomings exist in calculation of bearing capacity for design of shallow foundations. The conventional formulas to calculate the bearing capacity of shallow foundations are derived for uniform ground with cohesion, c and friction angle, ϕ based on the limit state theory of plasticity. However, if the ground below the foundation is not uniform or changes with the depth as in many practical situations, the formulas cannot be readily applied (Brown and Meyerhof, 1969). To overcome this limitation, the method of slices used in slope stability has been successfully applied to calculate of the bearing capacity of strip footings on rather complicated inhomogeneous ground (Imaizumi and Yamaguchi, 1986). On the other hand, Cheng et al., 2013 used the slip line solution for a bearing capacity problem to determine inter-slice force function for a horizontal slope. To study the bearing capacity of concrete caisson on rubble mound, Terashi and Kitazume (1987) carried out a series of centrifuge model tests of a foundation on top of high mound subjected to eccentric and inclined loads. The experimental results obtained from the tests well agreed with the factor of safety calculated from the simplified Bishop's method, and hence S.B. method has been recommended in Japanese design standard for port and harbor structure (Oversea Coastal Development Institute of Japan, 2009). As far as authors aware of, practical method to calculate the bearing capacity for non-uniform ground is not established.

A new slip circle method of slices is proposed in this study. The coefficients of bearing capacity calculated by the new method are almost equivalent to those calculated from the formula of plasticity. Based on the case studies, it is shown that the factor of safety for slope stability calculated by the proposed method well agreed with the actual performance of earth structures. The bearing capacity calculated from the proposed slip circle method for sandy layer underlained by soft clay was fairly agreed with the results of a series of centrifuge model tests carried out by Okamura et al. (1997).

2. New slip circle method of slices

Fig. 2 shows force vector equilibrium diagrams for all forces acting on a slice as shown in Fig. 1 for two cases.

where W_i': effective weight of the slice, l_i: chord length of the slice, T_i: shear force along the failure arc, N_i': effective normal force on the failure arc, V_i: shear force on the side of the slice, E_i': effective normal force on the side of the slice, α_i: inclination of the slice and n:number of slices.

As shown in Fig. 2, the equilibrium conditions of forces in vertical and horizontal directions are given in Eq. (1), and Eq. (2) respectively.

$$W_i^{'}+\Delta V_i = T_i \sin \alpha_i + N_i^{'} \cos \alpha_i \tag{1}$$

$$\Delta E_i^{'} = T_i \cos \alpha_i - N_i^{'} \sin \alpha_i \tag{2}$$

where $\Delta V_i = V_{i+1} - V_i$, $\Delta E'_i = E'_{i+1} - E'_i$

The shear force along the failure arc can be written as in Eq. (3). Where, the cohesion c_i and the friction angle ϕ_i are shear strength properties at the failure plane, and F_s is the factor of safety.

$$T_i = \left(c_i l_i + N_i^{'} \tan \phi_i\right)/F_s \tag{3}$$

Considering the equilibrium of moment of failure mass, the Eq. (4) can be obtained.

$$\sum T_i = \sum W_i^{'} \sin \alpha_i \tag{4}$$

The assumptions on resultant forces acting on the sides of the slices in M.F. and S.B. methods are given by Eq. (5), and Eq. (6) respectively.

$$\text{M.F. method}: \frac{\Delta V_i}{\Delta E_i^{'}} = \tan \alpha_i \tag{5}$$

$$\text{S.B. method}: \frac{\Delta V_i}{\Delta E_i^{'}} = 0 \quad (\Delta V_i = 0) \tag{6}$$

It is well known that in both modified Fellenius's and Bishop's methods, the equilibrium of forces on the slices is not satisfied. Morgenstern and Price (1965) showed a method that all the conditions on equilibrium of forces and moment are satisfied. In Morgenstern and Price method, the resultant forces acting on the sides of the slices are assumed as follows and are given in the following equation:

$$\text{Morgenstern} - \text{Price's Method} \ \frac{\Delta V_i}{\Delta E_i^{'}} = \lambda f(x) \tag{7}$$

where λ is a ratio of inter-slice shear and normal forces and $f(x)$ is a specified side force function. Spencer (1967, 1973) proposed a new equation to solve the slip circle of slices as given in the following equation:

$$\text{Spencer's Method} \ \frac{\Delta V_i}{\Delta E_i^{'}} = \tan \theta_0 \ (\theta_0 \ \text{isconstant}). \tag{8}$$

It is known that Spencer's method is equivalent to Morgenster–Price's method and the method satisfies the equilibrium of forces and moments on the slices (Fredlund and Krahn, 1977; Kondo and Hayashi, 1997). However, the assumption in Spencer's method, that θ_0 is constant for all the slices, does not seems to be reasonable when the sliding circle passes well below the base of the ground.

In this study, a new assumption is introduced as given in the following equation:

$$\text{Proposed method}: \frac{\Delta V_i}{\Delta E_i^{'}} = \tan(\beta \alpha_i) \tag{9}$$

where β is a parameter determining the relation between the inclination of failure arc and the direction of resultant forces acting on the side of each slice. As shown in Eqs. (5) and (6), $\beta=1$ for M.F. method and $\beta=0$ for S.B. method. In the proposed method, $\beta=0.250$ is used considering the consistency with bearing capacity factors.

Eq. (10) can be obtained by substituting ΔV_i from Eq. (9) into Eq. (1)

$$W_i^{'}+\Delta E_i^{'}\cdot\ \tan(\beta\alpha_i)=T_i\cdot\ \sin\ \alpha_i+N_i^{'}\ \cos\ \alpha_i\Delta E_i^{'}\cdot$$
$$=\frac{T_i\cdot\ \sin\ \alpha_i+N_i^{'}\ \cos\ \alpha_i-W_i^{'}}{\tan(\beta\alpha_i)} \tag{10}$$

Using Eq. (10) and Eq. (2), Eq. (11) can be written as follows:

$$W_i^{'}+\left(T\ \cos\ \alpha-N_{\rm i}^{'}\ \sin\ \alpha\right)\ \tan(\beta\alpha_i)=T_i\ \sin\ \alpha_i+N_i^{'}\ \cos\ \alpha_i \tag{11}$$

From Eq. (11), N'_i can be obtained as given in Eq. (12).

$$N_i^{'}=\frac{W_i^{'}+\left\{\cos\ \alpha_i\cdot\ \tan(\beta\alpha_i)-\ \sin\ \alpha_i\right\}T_i}{\sin\ \alpha_i\cdot\ \tan(\beta\alpha_i)+\ \cos\ \alpha_i} \tag{12}$$

Substituting Eq. (3) into Eq. (12), T_i is eliminated and N_i' is obtained as in Eq. (13)

$$N_i^{'}=\frac{W_i^{'}/\ \cos\ \alpha_i+c_il_i\left\{\tan(\beta\alpha_i)-\ \tan\ \alpha_i\right\}/F_s}{x_i-\left\{\tan(\beta\alpha_i)-\ \tan\ \alpha_i\right\}\tan\ \varphi_i/F_s} \tag{13}$$

where $x_i\ =1+\tan\alpha_i\times\tan(\beta\alpha_i)$

$$N_i^{'}=\frac{W_i^{'}/\ \cos\ \alpha_i+c_il_i\left\{\tan(\beta\alpha_i)-\ \tan\ \alpha_i\right\}/F_s}{1+\ \tan\ \alpha_i\cdot\ \tan(\beta\alpha_i)-\left\{\tan(\beta\alpha_i)-\ \tan\ \alpha_i\right\}\tan\ \varphi_i/F_s}.$$

From Eqs. (3), (4), and (9), N_i can be eliminated and F_s is obtained as shown in Eq.(14)

$$F_s=\sum\frac{x_i\cdot c_i\cdot l_i+\left(W_i^{'}/\ \cos\ \alpha_i\right)\tan\ \varphi_i}{x_i+\left\{\tan\ \alpha_i-\ \tan(\beta\alpha_i)\right\}\tan\ \varphi_i/F_s}\cdot\frac{1}{\sum W_i\ \sin\ \alpha_i}. \tag{14}$$

In Eq.(13), F_s is in both side of the equation and therefore similar procedure must be adopted in determining the factor of safety as for S.B. method. In the case for $\beta=1$, Eq. (12) becomes the equation for M.F. method and is derived as in Eq. (15a)

$$F_s=\sum\frac{c_i\cdot l_i+W_i^{'}\ \cos\ \alpha_i\cdot\ \tan\ \varphi_i}{W_i\ \sin\ \alpha_i}. \tag{15a}$$

In the case for $\beta=0$, Eq. (12) becomes the equation for S.B. method as presented in Eq. (15b)

$$F_s=\sum\frac{c_i\cdot l_i+\left(W_i^{'}/\ \cos\ \alpha_i\right)\tan\ \varphi_i}{1+\ \tan\ \alpha_i\cdot\ \tan\ \varphi_i/F_s}\cdot\frac{1}{\sum W_i\ \sin\ \alpha_i} \tag{15b}$$

Eq. (14) is applicable for the calculation of bearing capacity of strip footing. To apply the slip circle method to calculate the bearing capacity of rectangular or circular footings, three-dimensional effect must be taken into consideration for better estimation of the factor of safety. Three dimensional slope stability analysis methods, based on limit equilibrium or variation calculus, have been proposed by several researchers since 1970s (Baligh and Azzouz, 1975; Hovland, 1977). A majority of these methods assumed a symmetrical plane for the failure mass in order to eliminate the statistically indeterminate conditions (Chen and Chameau, 1982; Xing, 1988; Leshchinsky and Huang, 1992; Huang et al., 2002; Yamagami and Jiang, 1997; Jiang and Yamagami, 2004). In some of the remaining methods (e.g those of Ugai (1985), Hunger (1987), Lam and Freduland (1993)) a symmetrical plane was not assumed in the limit equilibrium formulation and those, in general, are extension of 2D theories such as Bishop, Janbu, Spencer and Sarma methods (Sarma, 1979).

A number of attempts have been made in calculation of bearing capacity and understanding the failure mechanisms of shallow foundations elsewhere. Nakase (1981) used an limit equilibrium method assuming cylindrical sliding surface for the analysis of bearing capacity for rectangular foundation. Narita and Yamaguchi (1992) carried out 3D analysis of bearing capacity for square and rectangular footings based on the slice method assuming the sliding surface to be log spiral with the extension of 2D method of slices. This analysis concluded that the bearing capacity factors, specially for N_q and N_γ for greater length to width ratio of the footing, calculated from the proposed method showed higher values than those from 2D analysis. Dewaikar and Mohapatra (2003) proposed an analysis based on Prandtl's failure mechanism and concluded that the N_γ obtained by the proposed analysis is well agreed with available experimental values. Further, they added that the pole of the log spiral lays at the footing edge for general shear failure condition and the point of application of the passive thrust is strongly influenced by friction angle of the soils. A new concept for the calculation of bearing capacity factors for shallow foundations was proposed based on classical Terzaghi-Buisman by Perau (1997) for all loading cases. Tani and Craig (1995) proposed a simple method to calculate the un-drained bearing capacity for a circular foundation under plain strain and axisymmetric conditions and compared the results with a series of centrifuge model tests conducted. Bearing capacity estimated from theoretical formulae was found to be greater (6–17%) than that of model tests and the failure mode, i.e. punching shear was accordance with both theoretical and experimental results. However, as far as authors aware of, none of these methods considered the multilayered ground or ground consisting of sand layer in determining the bearing capacity or its failure mechanism.

In this study, a circular cylindrical shaped sliding body is assumed as shown in Fig. 3. L and B are the length and the width of the foundation respectively.

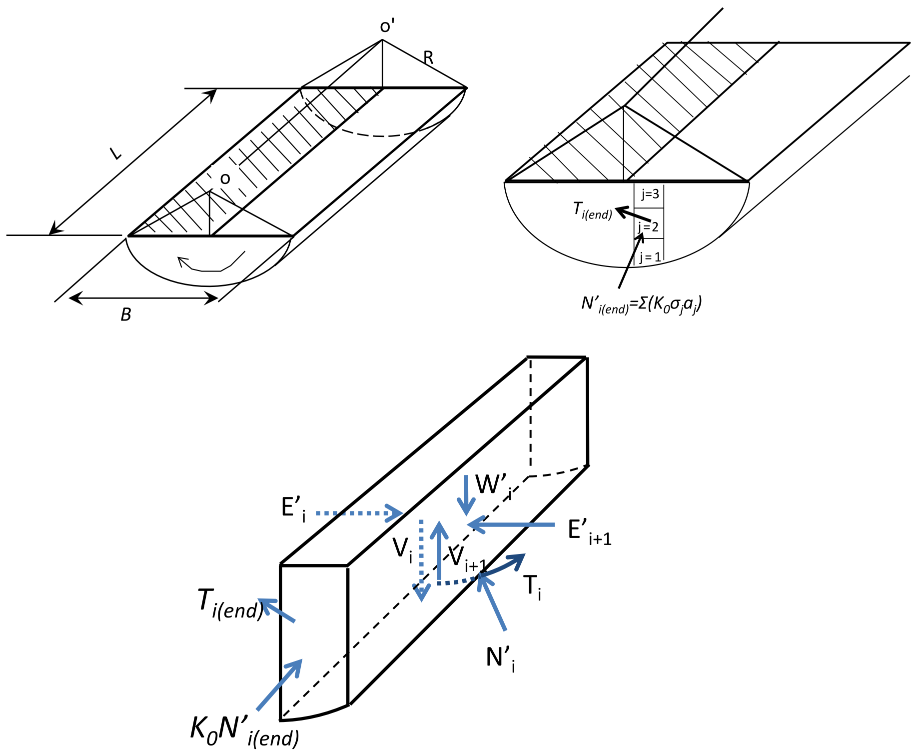

Fig. 3. Circular cylindrical shape sliding body and forces acting on a slice.

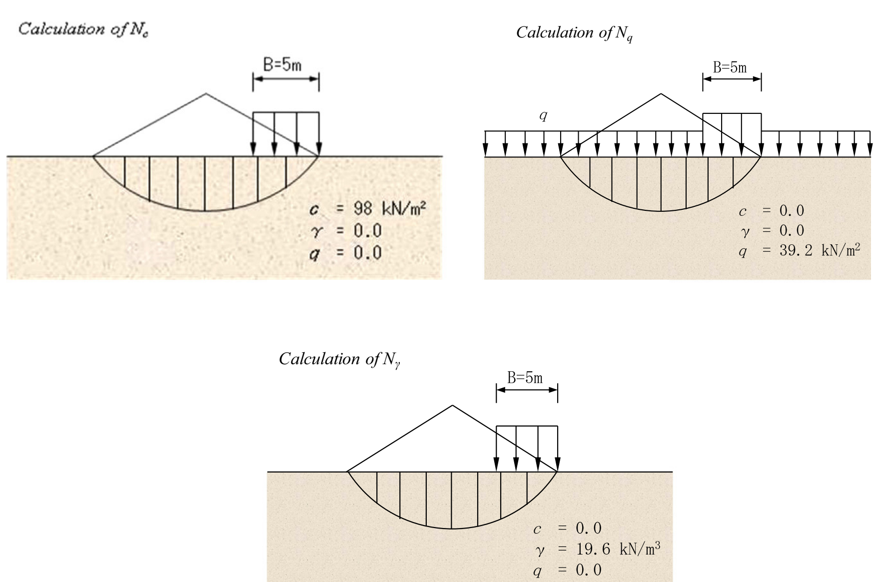

Fig. 4. Model for calculating bearing capacity factors by slip circle method.

The shear force acting on the two ends of the cylinder is obtained in Eq. (16) as follows:

$$T_{i(end)} = \sum_j \left(\frac{\sigma_j K_0 \tan \varphi_j + c_j}{F_s} a_j \right) \tag{16}$$

where K_0 is the coefficient of earth pressure at rest, and σ_j, ϕ_j, c_j and a_j are effective overburden stresses, friction angle, cohesion and the area of a slice, respectively.

Considering the equilibrium conditions of forces in vertical direction, Eq. (17) can be obtained

$$W_i' + \Delta V_i - \frac{2}{L} \left\{ \sum_j \left(\frac{\sigma_j K_0 \tan \varphi_j + c_j}{F_s} a_j \right) \right\} \sin \alpha_i = T_i \sin \alpha_i + N_i' \cos \alpha_i \tag{17}$$

where a_i is the area of the slice.

Eq. (18) shows the forces resolving in horizontal direction

$$\Delta E_i' - \frac{2}{L} \left\{ \sum_j \left(\frac{\sigma_j K_0 \tan \varphi_j + c_j}{F_s} a_j \right) \right\} \cos \alpha_i = T_i \cos \alpha_i + N_i' \sin \alpha_i. \tag{18}$$

From the equilibrium of moments, Eq. (19) can be obtained

$$L \sum R T_i + 2 \left\{ \sum_j \left(\frac{\sigma_j K_0 \tan \varphi_j + c_j}{F_s} a_j r_j \right) \right\} = LR \sum W_i \sin \alpha_i \tag{19}$$

where R is a radius of the circle, and r_j is a distance of a slice at both ends from the center of circle O–O' as shown in Fig. 3. Using Eqs. (9), Eq. (17) can be deduced to Eq. (20) as

$$W_i' + \Delta E_i' \cdot \tan(\beta \alpha_i) - \frac{Y_0 \sin \alpha_j}{F_s} = T_i \sin \alpha_i + N_i' \cos \alpha_i \tag{20}$$

where $Y_0 = \frac{2}{L} \sum_j \left(\sigma_j K_0 \tan \varphi_j + c_j \right) a_j$

Eq. (18) can be rewritten as in Eq. (21),

$$\Delta E_i' - \frac{Y_0 \cos \alpha_j}{F_s} = T_i \cos \alpha_i - N_i' \sin \alpha_i \tag{21}$$

Using, $Y_1 = 2 \sum_j \left(\sigma_j K_0 \tan \varphi_j + c_j \right) a_j r_j$ Eq. (19) can be written as

$$\sum T_i + \frac{Y_1}{LRF_s} = \sum W_i \sin \alpha_i. \tag{22}$$

From Eqs.(20) and (21), the N'_i is obtained and are presentend in Eq. (23).

$$N_i' = \frac{W_i' + \{ \cos \alpha_i \cdot \tan(\beta \alpha_i) - \sin \alpha_i \} \cdot T_i + Y_0 \cos \alpha_i / F_s \tan(\beta \alpha_i) - Y_0 \sin \alpha_i / F_s}{\sin \alpha_i \cdot \tan(\beta \alpha_i) + \cos \alpha_i} \tag{23}$$

Eliminating T_i from Eq. (3), N'_i is given as in Eq. (24) as follows:

$$N_i' = \frac{W_i' + \{ \cos \alpha_i \cdot \tan(\beta \alpha_i) - \sin \alpha_i \} \cdot (c_i l_i + N_i \tan \varphi_i)/F_s + Y_0 \cos \alpha_i / F_s \tan(\beta \alpha_i) - Y_0 \sin \alpha_i / F_s}{\sin \alpha_i \cdot \tan(\beta \alpha_i) + \cos \alpha_i} \tag{24}$$

As Eq. (24) contains N_i in both sides, N'_i is rewritten in Eq. (25) as follows:

$$N_i' = \frac{W_i' / \cos \alpha_i + c_i l_i \{ \tan(\beta \alpha_i) - \tan \alpha_i \} / F_s + Y_0 \cos \alpha_i \tan(\beta \alpha_i) - Y_0 \sin \alpha_i / F_s \cos \alpha_i}{1 + \tan \alpha_i \cdot \tan(\beta \alpha_i) - \{ \tan(\beta \alpha_i) - \tan \alpha_i \} \tan \varphi_i / F_S} \tag{25}$$

From Eqs. (3) and (22), Eq. (26) can be obtained

$$\sum \frac{c_i l_i + N_i \tan \varphi_i}{F_S} + \frac{Y_1}{F_S RL} = \sum W_i \sin \alpha_i. \tag{26}$$

Using Eqs. (25) and (26), F_s can be written and are shown in Eq. (27)

$$F_S = \left\{ \sum \left(\frac{x_i \cdot c_i l_i + \frac{W_i'}{\cos \alpha_i} \tan \varphi_i + \left(\frac{Y_0 \cos \alpha_i \tan(\beta \alpha_i) - Y_0 \sin \alpha_i}{F_S \cos \alpha_i} \right) \tan \varphi_i}{x_i + \frac{\{ \tan \alpha_i - \tan(\beta_i \alpha_i) \} \tan \varphi_i}{F_S}} \right) + \frac{Y_1}{RL} \right\} \cdot \frac{1}{\sum W_i \sin \alpha_i} \tag{27}$$

where $\chi_i = 1 + \tan \alpha_i \tan(\beta \alpha_i)$.

Rearranging the terms in Eq. (27), the factor of safety is deduced to the following form as in Eq. (28):

$$F_S = \left\{ \sum \left(\frac{c_i l_i + \frac{W_i' \cos \beta\alpha_i}{\cos(\alpha_i - \beta\alpha_i)} \tan \varphi_i + \left(\frac{Y_0 \cos \alpha_i \sin(\beta\alpha_i) - Y_0 \sin \alpha_i \cos \beta\alpha_i}{F_S \cos(\alpha_i - \beta\alpha_i)} \right) \tan \varphi_i}{1 + \frac{\tan(\alpha_i - \beta_i\alpha_i) \tan \varphi_i}{F_S}} \right) + \frac{Y_1}{RL} \right\} \cdot \frac{1}{\sum W_i \sin \alpha_i}$$

$$= \left\{ \sum \left(\frac{c_i l_i + \frac{W_i' \cos \beta\alpha_i}{\cos(\alpha_i - \beta\alpha_i)} \tan \varphi_i - \frac{Y_0 \tan(\alpha_i - \beta\alpha_i) \tan \varphi_i}{F_s}}{1 + \frac{\tan(\alpha_i - \beta_i\alpha_i) \tan \varphi_i}{F_S}} \right) + \frac{Y_1}{RL} \right\} \cdot \frac{1}{\sum W_i \sin \alpha_i} \quad (28)$$

where

$Y_0 = \frac{2}{L} \sum_{j=1} \left(\sigma_j K_0 \tan \varphi_j + c_j \right) a_j, Y_1 = 2\sum_j \left(\sigma_j K_0 \tan \varphi_j + c_j \right) a_j r_j.$

3. Bearing capacity factors in plane strain condition calculated based on slip circle method

In general the formula for ultimate bearing capacity of shallow foundation is given in Eq. (29) as follows:

$$q_{ult} = N_c c_0 + \gamma_2 D N_q + \frac{\gamma_1 B}{2} N_\gamma \quad (29)$$

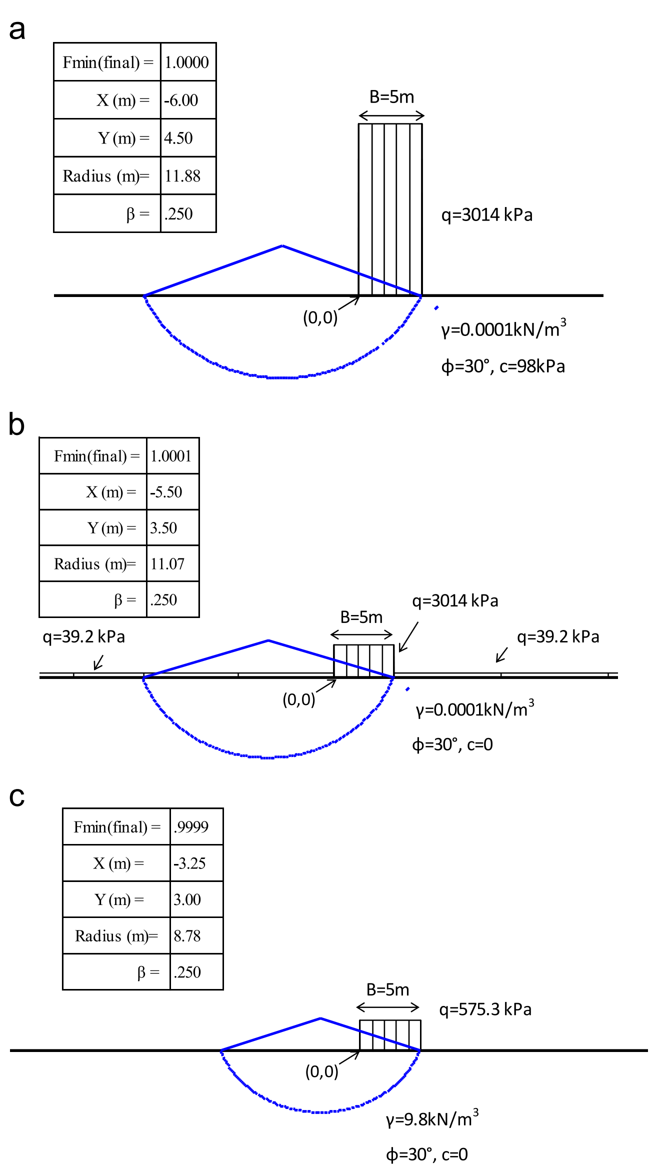

Fig. 5. Examples of calculation of bearing capacity factor by circle slide method.

where q_{ult}: ultimate bearing capacity of foundation, c: cohesion of soil below the foundation, B: width of the foundation, D: depth at the bottom of the foundation, γ_1: unit weight of soil below the foundation, γ_2: unit weight of soil above the foundation, and N_c, N_γ, N_q: bearing capacity factors.

When the unit weight, cohesion, and friction angle of soil are uniform, the factors N_q and N_c are theoretically derived based on the plastic analysis by Reisnner and Prandtl, as shown in Eq. (30) and Eq. (31) respectively (Vesic, 1975)

$$N_q = \frac{1+\ \sin\ \varphi}{1-\ \sin\ \varphi}\exp(\pi\ \tan\ \varphi) \tag{30}$$

$$N_c = \left(N_q - 1\right)\cot\varphi \tag{31}$$

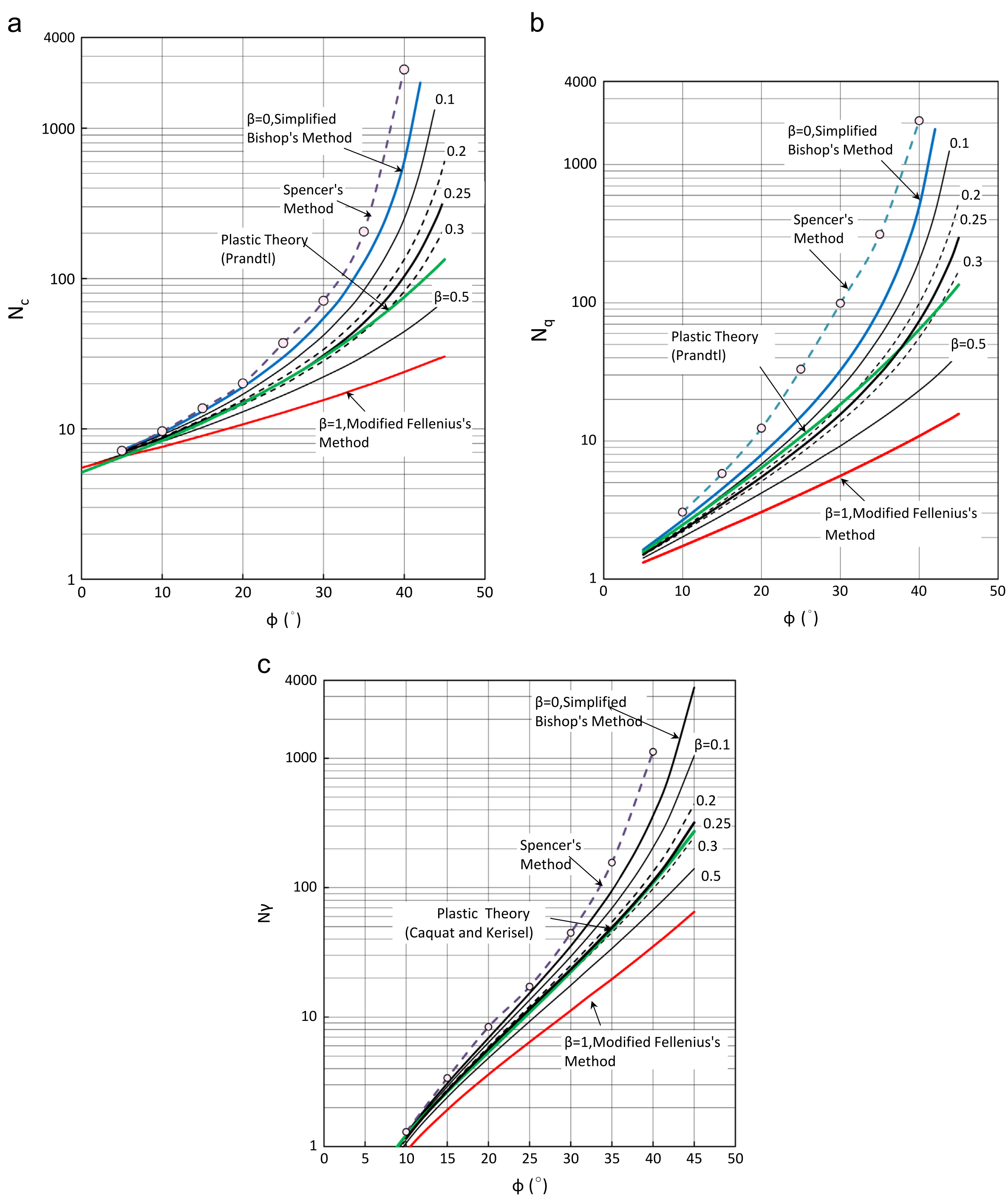

Fig. 6. (a) N_c and ϕ. (b) N_q and ϕ. (c) N_γ and ϕ.

For the factor N_γ, Vesic showed Eq. (32) for approximating the values of N_γ numerically evaluated by Caquot and Kerisel (Caquot and Kerisel, 1953; Vesic, 1975)

$$N_\gamma = 2\left(N_q + 1\right) \tan \varphi \tag{32}$$

Over the years, these equations have been commonly used for design in geotechnical engineering structures.

In the practice, unit weight or shear strength parameters below the foundation are not uniform, thus, the design engineers are often worried how to represent a complicated ground to a uniform ground so as to use the above equations for calculations. The slip circle method of slices is a useful tool to calculate the bearing capacity of the complicated ground. Nakase (1981) calculated a set of bearing capacities of clay layer that increases strength linearly with the depth, from slip circle method of slices.

3.1. Some reviews on ultimate bearing capacity based on slip circle method

The bearing capacity factor N_c for clay ($\phi = 0$) calculated from slip circle method of slices is 5.52 while that of plastic theory is 5.14. Hence, the bearing capacity factor N_c was found to be greater in value when using the slip circle method of slices than that of plastic solution. As the difference of 7% is generally allowable in the geotechnical engineering designs, the slip circle method is practically applicable to clayey ground. However, when the ground consists of sand $\phi > 0$, it is well known that the bearing capacity calculated from slip circle method shows considerable difference with the plastic solution, and that in M.F. method, the bearing capacity is considerably smaller than the plastic solution, while in S.B. method too large values are calculated (Yamaguchi, 1984).

Imaizumi and Yamaguchi (1986) used a failure surface consists of an arc and a straight line with consideration of the active edge, and showed that the differences with the Prandtl's solution can be minimized for the uniform sandy ground. However, to apply this method into practice, determination of the point where the sliding surface transfers from arc to straight line is problematic. In this study, a new assumption, as expressed in Eq. (8), was introduced and Eq. (14) is derived. The ultimate bearing capacity values were calculated from Eq. (14) for different values of β for a uniform level ground. Fig. 4 shows the ground condition used for the circle slip analysis to calculate the bearing capacity. For the calculation of N_c, the uniform load along 5 m width on the horizontal ground which has a cohesion of 98 kPa and friction angle φ was assumed. The slip circle passes one of the ends of uniform load and the minimum factor of safety was searched. The bearing capacity is the load when the minimum factor of safety is 1.000 ± 0.001. Figs. 5(a), (b) and (c) show the examples of the results of calculation to obtain N_c, N_q and N_γ, respectively, when the minimum factor of safety becomes to 1.000 ± 0.001 for the ground having $\varphi = 30^\circ$. Based on the calculated bearing capacity, the bearing capacity factors N_c, N_q, and N_γ were obtained.

3.2. Bearing capacity calculated from the proposed slip circle method of slices

Figs. 6(a), (b) and (c) illustrate the variation of the bearing capacity factors, N_c, N_q and N_γ with friction angle, ϕ, respectively. Prandtl's solutions given in Eqs. (30), (31), and (32) are also plotted in the same graph for the comparison. As seen in Fig. 6(a), (b), and (c), the bearing capacity factors calculated from slip circle method of slices decrease as β varies from 0 (S.B. method) to 1(M.F. method). The bearing capacity factors calculated from M.F. method are considerably lesser than that of Prandtl's solution. In contrast, those calculated from S.B. method showed significantly greater values. When β ranging from 0.2 to 0.3, the bearing capacity factors calculated are consistent with Prandtl's solution. In Fig. 6(a), (b), and (c), the values of N_c, N_q and N_γ calculated by Spencer's method are also plotted. As shown in the figures, the values are much larger than Prandtl's solutions, meaning that the assumption of Eq. (8) is not appropriate to determine the bearing capacity for a horizontal ground.

Fig. 7 shows the difference of N_c, N_q and N_γ calculated by circular slip method from Prandtl's solutions with β ranging from 0.1 to 0.4, where the difference was shown as in the following equation:

$$\sum_{\varphi = 5^\circ}^{\varphi = 40^\circ} \frac{\left(N_{CBCF} - N_{Theory}\right)}{N_{Theory}} \tag{33}$$

where N_{CBCF} and N_{Theory} are the bearing capacity factors calculated by slip circle method and Eqs. (30), (31) and (32) of Prandtl's solution and ϕ is the friction angle. Fig. 7 shows that the difference is smaller between 0.2 and 0.3 of β, and the average of differences of 3 coefficients is zero when $\beta = 0.25$. In this study, $\beta = 0.25$ in Eq. (10) is proposed and the calculated bearing capacity factors and the Prandtl's solution are listed in Table 1. As shown in Table 1, the coefficients of bearing capacity calculated by proposed slip circle method fairly agreed with Prandtl's solution. in the rangr of $\phi \leq 40^\circ$. Especially, as for the value of N_γ which is the most governing factor in most of the geotechnical practices, the difference is within 5% between the proposed method and Prandtl's solution. Hereafter, the authors refers the proposed method with $\beta = 0.25$ as "CBCF method", where CBCF stands for the circle bearing capacity factor. As shown in Table 1, when ϕ is greater than 40°, CBCF method overestimates the bearing capacity factors considerably and thus the proposed method is successfully applied for the soils having friction angle less than 40°.

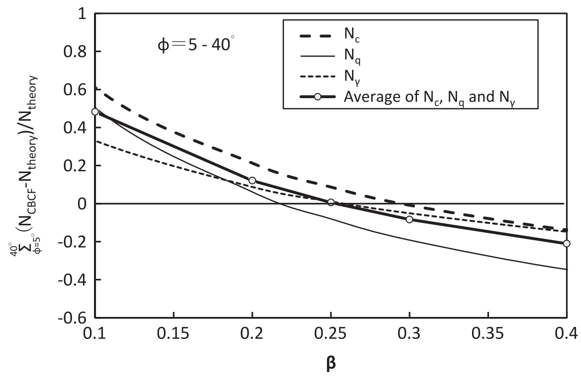

Fig. 7. Comparison of CBCF calculated by slip circle method of slices from conventional Eq.

Table 1
Comparison of bearing capacity factors (β=0.250).

ϕ	Proposed method (β=0.25)			Eqs. (30), (31) and (32)		
	N_c	N_q	N_γ	N_c	N_q	N_γ
0	5.5	1.0	0.0	5.1	1.0	0.0
5	7.0	1.5	0.4	6.5	1.6	0.4
10	8.7	2.3	1.2	8.3	2.5	1.2
15	11.3	3.5	2.7	11.0	3.9	2.6
20	15.0	5.5	5.7	14.8	6.4	5.4
25	20.9	9.0	11.5	20.7	10.7	10.9
30	30.8	15.6	23.5	30.1	18.4	22.4
32	37.0	20.1	31.4	35.5	23.2	30.2
34	45.4	26.5	42.4	42.2	29.4	41.1
36	57.5	35.5	57.6	50.6	37.7	56.3
38	75.0	50.0	79.7	61.3	48.9	78.0
40	103.5	74.0	112.4	75.3	64.2	109.4
42	153.0	117.0	164.4	93.7	85.4	155.5
44	252.0	205.0	250.8	118.4	115.3	224.6

3.3. Clarification of β=0.25 used in proposed CBCF method

The reason to use $\beta=0.25$ in determining the bearing capacity factors is discussed in this section. The factor of safety in CBCF method can be evaluated from Eq. (14). Substituting $x_i=1+\tan\alpha_i\tan(\beta\alpha_i)$ into Eq. (14), the modified form is as shown in the following equation:

$$F_s=\sum\frac{c_i\cdot l_i+\frac{\left(W_i'/\cos\alpha_i\right)\tan\varphi_i}{1+\tan\alpha_i(\tan\beta\alpha_i)}}{1+\frac{\tan\alpha_i-\tan(\beta\alpha_i)}{1+\tan\alpha_i(\tan\beta\alpha_i)}\tan\varphi_i/F_s}\cdot\frac{1}{\sum W_i\sin\alpha_i}$$

$$=\sum\frac{c_i\cdot l_i+\frac{\left(W_i'/\cos\alpha_i\right)\tan\varphi_i}{1+\tan\alpha_i(\tan\beta\cdot\alpha_i)}}{1+\tan\left\{(1-\beta)\cdot\alpha_i\right\}\tan\varphi_i/F_s}\cdot\frac{1}{\sum W_i\sin\alpha_i} \tag{34}$$

To get the stable solution, the factor of safety, F_s must satisfy the following condition:

$$F_s\rangle-\tan\left\{(1-\beta)\cdot\alpha_i\right\}\tan\varphi_i \text{ for all slices} \tag{35}$$

When F_s is equal to 1.0, i.e to calculate the bearing capacity, Eq. (35) deduces to Eq. (36)

$$-\tan\left\{(1-\beta)\cdot\alpha_i\right\}<\frac{1}{\tan\varphi_i}\text{ for all slices.} \tag{36}$$

Eq. (36) gives the constraint condition of $-\alpha_i$ to search the minimum factor of safety in circle slide method. In Simplified Bishop's method, i.e is $\beta=0$, the conditions of α_i for $\phi_\iota=30°$, 35° and 40° are given as follows:

$-\alpha_i<60°$ ($\phi_\iota=30°$), $-\alpha_i<55°$ ($\phi_\iota=35°$), $-\alpha_i<50°$ ($\phi_\iota=40°$)

When $\beta=0.25$ is used, the constraint conditions for $-\alpha_i$ are follows:

$-\alpha_i<80°$ ($\phi_\iota=30°$), $-\alpha_i<73.3°$ ($\phi_\iota=35°$), $-\alpha_i<66.7°$ ($\phi_\iota=40°$)

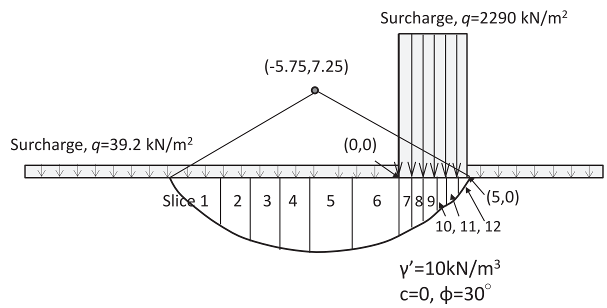

Fig. 8. An example for calculation of F_s under different values of β.

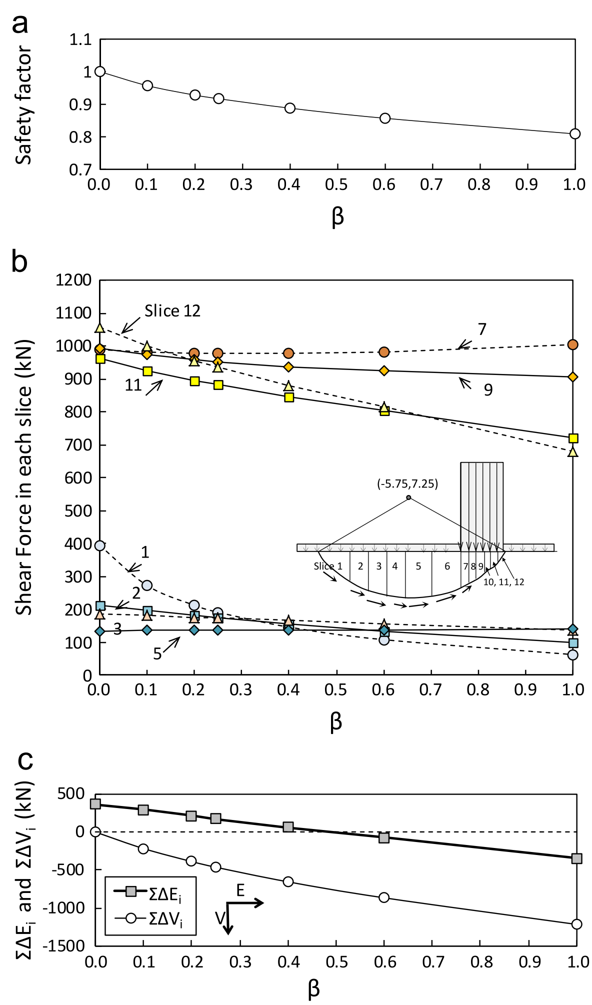

Fig. 9. Factor of safety and forces of individual slices with β. (a) factor of safety with β, and (b) shear forces of each slices with β. (c) Sumation of inter-slice horizontal forces $\Sigma\Delta E_i$ and vertical forces $\Sigma\Delta V_i$ of slices with β.

The present method, i.e is $\beta=0.25$, enables to search the minimum factor of safety in broaden range of $-\alpha_i$.

The stability analysis of horizontal ground was conducted for an example shown in Fig. 8 and values of factor of safety were calculated for β ranging from 0 to 1. The overall factor of safety calculated for different β values are shown in Fig. 9(a). Fig. 9(b) shows the shear forces of individual slices for different values of β. Considerable variation of the shear force observed in the slices at or near to the both ends of the failure circle for different β values. However, shear forces showed less variation for different β values of the slices away from the end of circular slip. Fig. 9(c) shows summation of inter-slice horizontal forces working in each slice, $\Sigma\Delta E_i$, and inter-slice vertical forces, $\Sigma\Delta V_i$. As shown in the figure, summation of all inter-slice vertical forces is less than zero except for $\beta=0$ (i.e for Simplified Bishop method). This leads to underestimates the factor of safety and it is more significant for higher values of β. The summation of inter-slice horizontal forces of all slices close to zero when β closes to 0.5. Also, as seen in Fig. 9(b), values of shear forces are higher when β is smaller. When $\beta=0.25$, both effects of horizontal and vertical inter-slice forces approach to a balancing condition. As for these reasons, $\beta=0.25$ is proposed in this study. Above discussion show some reasons for $\beta=0.25$ assumption, however, theoretical justification could not be made. To confirm the validity of CBCF method, the evidences based on experiments and case studies are discussed later in this paper.

4. Application to bearing capacity of dense sand layer overlying soft clay

In order to examine the applicability of proposed CBCF method, the results of series of centrifuge model loading tests conducted on dense sand overlaying soft clay by Okamura et al. (1997 and 1998) were used. Fig. 10 shows the setup of model circular and strip footings in the centrifuge.

4.1. Bearing capacity of strip footing

The test conditions of the centrifuge model tests are shown in Tables 2 and 3 for strip footing and circular footing, respectively (Okamura et al., 1998). The thickness of sand layer varies from 1.5 m to 6 m, and the relative density of sand was 88%. The undrained shear strength at the top of clay layer varied from 8.7 kPa to 86.7 kPa. Table 2 shows the results of the model tests for strip footing without embedment, and Table 3 shows the results of the model tests for circular footing without embedment. ($2q_f/\gamma'B$) in Tables 2 and 3 is normalized bearing capacity, q_f is the ultimate bearing capacity (stress) obtained in the experiments, γ' is submerged unit weight of sand layer, B is the width of the footing, and H is the thickness of the upper sand layer. According to Okamura et al. (1997), the bearing capacity q_f was defined as the peak load intensity for the load intensity – settlement curves in which the peak load was obtained, for the curves which do not show the peak load, q_f was determined at the intersection point of two straight lines extrapolated from the initial and final portions of the curve. As the settlement of model foundation at the bearing capacity was larger than 10% of B in almost all cases, the ground beneath the foundation seems to be in the condition of plastic failure.

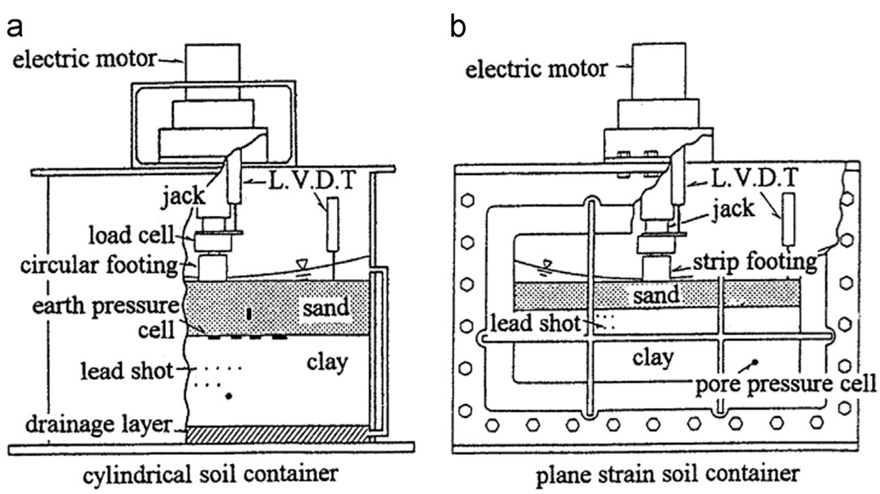

Fig. 10. Centrifuge model tests by Okamura et al. (1997).

Table 2
Condition and experimental result of the centrifuge model tests for strip footing on sand layer without embedment (SS series, Okamura et al., 1998).

Case	Width of the footing B (m)	Thickness of sand H (m)	H/B	Strength at clay surface c_0 (kPa)	Gradient of strength increase k[a] (kN/m^3)	Normalized bearing capacity, $2q_f/\gamma'B$: Experiments	Normalized bearing capacity, $2q_f/\gamma'B$: Calculated	Failure mode[a] (experiment)	Type of critical slip surface (calculated)
s-1	2.0	0.0	0.5	22.4	39	12.0	12.9	General shear	A
s-2		2.0	1.0	21.9	37	28.5	31.2	Punching shear	B
s-2		2.0	1.0	21.9	37	32.5	31.2		
s-3	1.5	3.0	2.0	22.4	39	78.8	80.9	Punching shear	B
s-4	1.0	3.0	3.0	22.4	40	136	169.4	Punching shear	B
s-5		4.0	4.0	23.0	40	222	203.2	General shear	A

[a]g = general shear, p = punching shear.

Table 3
Experimental value and calculated bearing capacity for circular footing (SC series, Okamura et al., 1998).

Case	Clay type	Preconsolidation Pres. of clay (kPa)	B (m)	H (m)	H/B	Normalized bearing capacity, $2q_f/\gamma' B$		Failure mode (experiment)	Type of critical slip surface (calculated)
						Experiment	Calculated		
c-1a	NC	60	3.0	1.5	0.5	10.2	9.7	Punching shear	B
c-1b				3.0	1.0	32.0	23.9	Punching shear	B
c-1c				4.5	1.5	70.8	50.3	Punching shear	B
c-1d				6.0	2.0	130	85.0	Punching shear	B
c-1e		30	1.5	1.5	1.0	36.9	32.3	Punching shear	B
c-1f				3.0	2.0	133	95.1	Punching shear	A
c-2a	OC08	78	3.0	0	0	10.0	14.6	General shear	A
c-2b				3.0	1.0	44.5	34.3	Punching shear	B
c-2c			2.0	3.0	1.5	90.3	69.5	Punching shear	B
c-2d			1.5	3.0	2.0	143	114.0	General shear	A
c-3a	OC2	192	3.0	1.5	0.5	28.9	30.4	Punching shear	B
c-3b				3.0	1.0	58.3	54.3	Punching shear	B
c-3c				4.5	1.5	103	85.4	Punching shear	B
c-3d				6.0	2.0	125	125.8	General shear	A
c-3e			1.5	1.5	1.0	83.0	93.0	Punching shear	B
c-3f				3.0	2.0	140	113.1	General shear	A
c-3g				4.5	3.0	147	112.7	General shear	A
c-4a	OC4	392	3.0	3.0	1.0	73.3	86.5	Punching shear	B
c-4b				4.5	1.5	141	106.7	Punching shear	A
c-4c			1.5	4.5	3.0	148	115.1	General shear	A

The bearing capacity values for the cases of model tests were calculated from the proposed CBCF method (Eq. (14)) with the increments of the vertical load (as surcharge) until the safety factor becomes 1.000 ± 0.001. According to Okumura et al., the bearing capacity increases with the increase of thickness of sand layer to width of the footing ratio, *H/B* until it reached that of uniform sand. Okamura et al. examined the bearing capacity based on their model with $\phi'=47^{\circ}$ for strip footing and $\phi'=40^{\circ}$ for circular footing for their analysis, considering the relative density of Toyoura sand (90% and 86%) used in the model test. These friction angles were obtained from the peak deviator stress of plane strain triaxial compression test and axisymmetrical triaxtial compression test carried out by Fukushima and Tatsuoka (1984) and Tatsuoka et al. (1986). However, Tatsuoka et al. (1992) pointed out that, to calculate the bearing capacity of horizontal sandy ground with conventional equations based on plasticity analysis, the friction angle must be determined with consideration of various factors such as strength anisotropy, stress dependency of ϕ', sample disturbances or effects of strain softening. According to Tatsuoka et al., to determine the friction angle from the peak deviator stress at the triaxial tests is not appropriate for using conventional equations of bearing capacity factors. In this study, the effective internal friction angle of the upper layer of sand was determined from the maximum value of measured bearing capacity in the model tests as $\phi'=43^{\circ}$ for the strip footing (plane strain condition) and $\phi'=41.5^{\circ}$ for the circular footing (axisymmetric condition) by back-calculating the bearing capacity with the conventional formulas as in Eqs. (30), (31) and (32) for uniform sand.

The values of normalized bearing capacity, $2q_f/\gamma' B$ calculated from the CBCF method are shown in Table 2 for strip footing. Fig. 11 shows the variation of normalized bearing capacity, $2q_f/\gamma' B$ with *H/B* ratios for strip footing. The normalized bearing capacity found in the modeled experiments was compared with that evaluated from the CBCF method. It is illustrated in Fig. 12 As shown in Figs. 11 and 12 the bearing capacity values that obtained in the modeled experiments are found to be agreed with that calculated from CBCF method within $\pm 25\%$ error. In the figure, the accuracy of the calculation in the range of $2q_f/\gamma' B$ from 100 to 150 is not high. As this range is a point that the failure mode changes from punching shear failure to general shear failure, the low accuracy may be due to the change of failure type. Within the limitation of 25% error, the proposed CBCF method seems to be successfully applied for determination of bearing capacity for dense sand layer overlying soft clay with different thicknesses.

Fig. 13 demonstrates the critical slip surfaces which showed the minimum factor of safety in the proposed CBCF analysis. In type A, the slip surface is inside the sand layer passes through the edge of the footing, while in type B, the slip surface passes close to the boundary between sand and the clay just below the edge of the footing as marked 'X' in Fig. 13. In Table 2, the failure mode in the experiments and the type of the critical slip surface are shown. As shown in Table 2, when the failure mode is general shear, the critical slip surface is type A, passing thorough the edge of the footing, while in the case of punching shear, the critical slip surface in CBCF method is type B, passing through the boundary between sand and clay layers just below the edge of the footing.

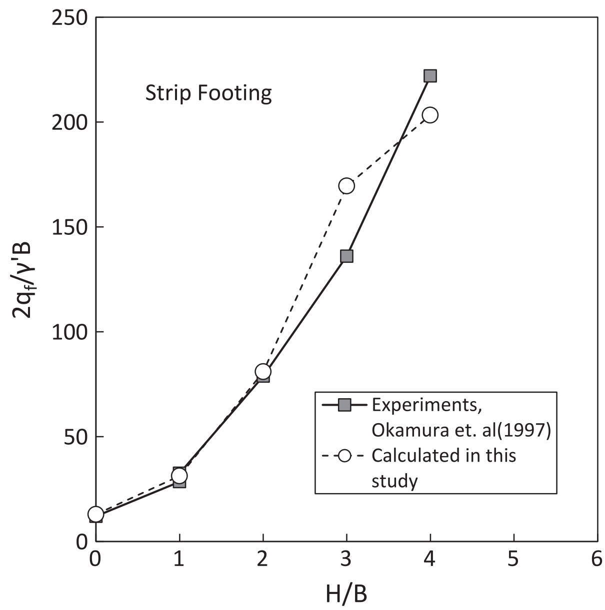

Fig. 11. Normalized bearing capacity and ratio of thickness of sand to width of the footing (strip footing).

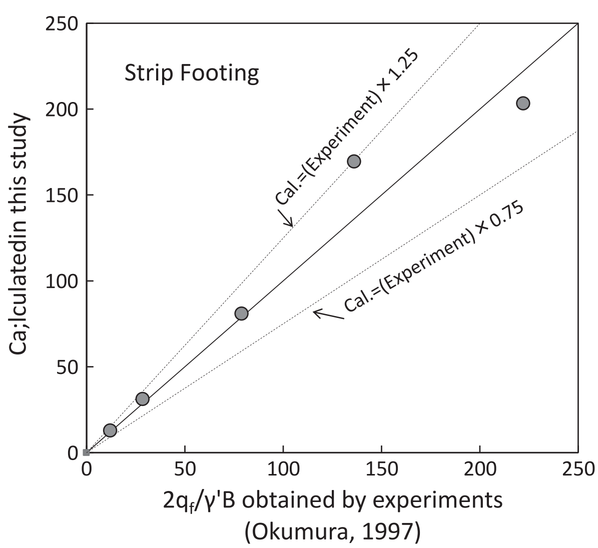

Fig. 12. Comparison of normalized bearing capacity for strip footing.

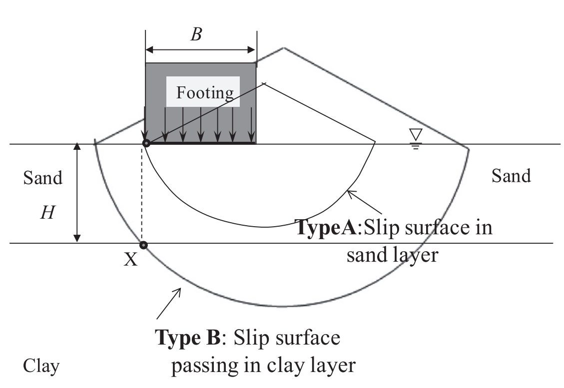

Fig. 13. Critical sliding surfaces for CBCF method.

4.2. Bearing capacity of circular footings

The bearing capacity factors of the circular footing was calculated as that of square footing with the same area using Eq.(28), where the value of K_0 is 0.5. Fig. 14 illustrates the normalized bearing capacity, $2q_f/\gamma' B$ varying with H/B ratio for different width of circular footing. In calculating the bearing capacity, the circular footing is replaced with the square footing with the same area. In Fig. 15 the normalized bearing capacity that obtained under model tests and that calculated from CBCF method are compared. As shown in Figs. 14 and 15 the experimental and the calculated bearing capacity values almost agreed with that calculated from CBCF method within $\pm 25\%$ errors. In the figures, it seems that CBCF method underestimates the bearing capacities for the cases of soft clay (C-1) and, conversely, overestimates for cases of relatively stiff clay (C-3 and C-4). The reason for this difference cannot be made clear in this study and it will be another limitation of CBCF method. Allowing 25% errors, it seems that the CBCF method can be used to estimate the bearing capacity of dense sand layer overlying soft clay with different thicknesses. Comparing the failure mode with the type of critical slip surface, as similar with the strip footing, general shear and punching shear correspond to type A and type B, respectively, except the cases for c-1f and c-4b where the difference of bearing capacities of type A and type B were very small.

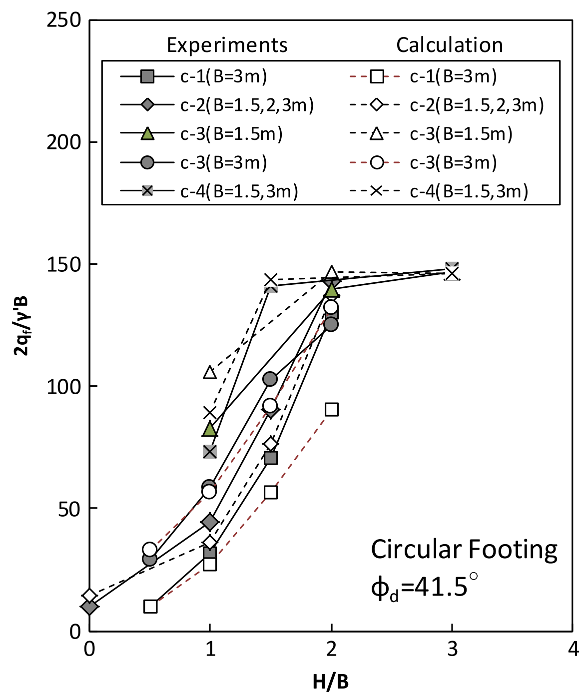

Fig. 14. Variation of normalized bearing capacity for different H/B ratios (circular footing).

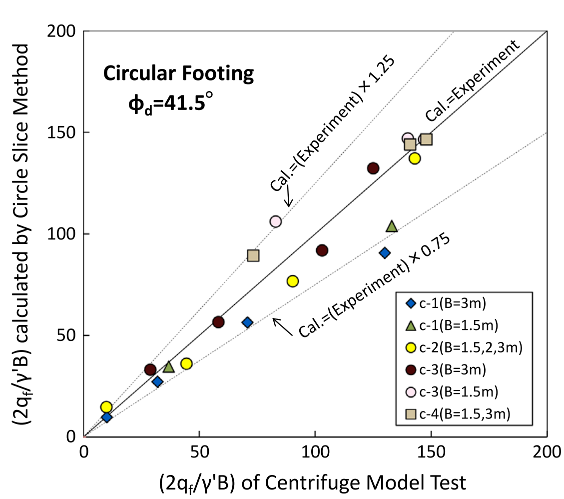

Fig. 15. Comparison of normalized bearing capacity for circular footing.

5. Use of CBCF method for slope stability analysis of soft gound consisting of sand and clay layers

Kobayashi (1984) carried out two case studies (Cases A and B in this study) on the stability of a ground consisting of clay layer overlain by sand layer showing that the factor of safety calculated by conventional Modified Fellenius's method underestimates in both cases. Fig. 16 shows a situation of large oil tank constructed on a reclaimed land, which is filled with the water for a loading test (Case A). The factor of safety calculated from M.F., S.B., and CBCF methods were 0.734, 1.590, and 1.269, respectively. As the loading test of the oil tank was carried out safely, it was apparent that the factor of safety calculated from M.F. method was too small. Kobayashi carried out a finite element stability analysis by strength reduction technique, where the Mohr–Coulumb's failure criteria and the elasto-plastic model was used for soils (Kobayashi, 1984; Matsui and San, 1992). The factor of safety obtained by the finite element method was 1.17 and is close to 1.269 by CBCF method.

Fig. 17 shows Case B of a breakwater constructed on a double-layered ground (Kobayashi, 1984). The factor of safety calculated from M.F., S.B. and CBCF methods were 0.914, 1.633 and 1.261, respectively. The concrete caisson was placed on the rubble mound safely and the caisson was filled with soil. After the placement and filling, although about 80 cm settlement took place during 150 days, the structure was in stable condition, which suggested that the factor of safety is greater than 1, but the margin of the safety would not be so high. Among the factors of safety calculated from three methods, that evaluated from proposed CBCF method seems to be reasonable for explaining the actual behavior of the structure. Kobayashi (1984) showed that the factor of safety obtained from the finite element stability analysis by strength reduction technique was 1.36, which was close to 1.261 obtained under CBCF method.

Fig. 18 shows a stock yard of iron ore in reclaimed land, where Holocene marine clay at the seabed was improved with sand drain method and the reclaimed soil was pure sand having internal friction angle $\phi = 30°$(Saito, 1977). The factors of safety calculated were 0.916 from M.F. method, 1.807 from S.B. method and 1.422 from CBCF method. As the stock of iron ore was carried out safely without any problems on stability of ground, this case also showed that the factor of safety obtained from M. F. method was significantly low.

The factors of safety for cases A, B, and C are listed in Table 4 for the comparison. As shown in Table 4, CBCF method gives the intermediate factor of safety between M.F. and S.B. methods and as mentioned above, the factor of safety obtained by CBCF method seems to be consistent with actual behavior of the ground consisting of clay layer overlain by sand layer.

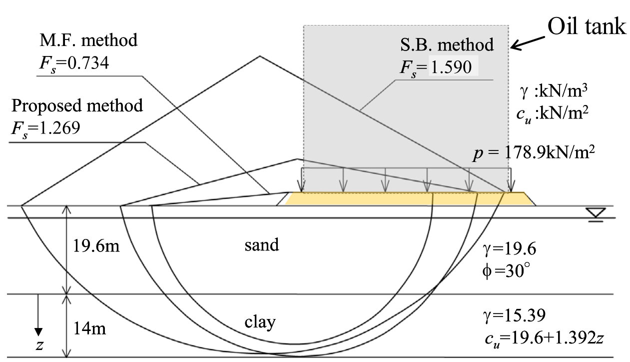

Fig. 16. Cross section and critical slip circles for *Case A* (Kobayashi, 1984).

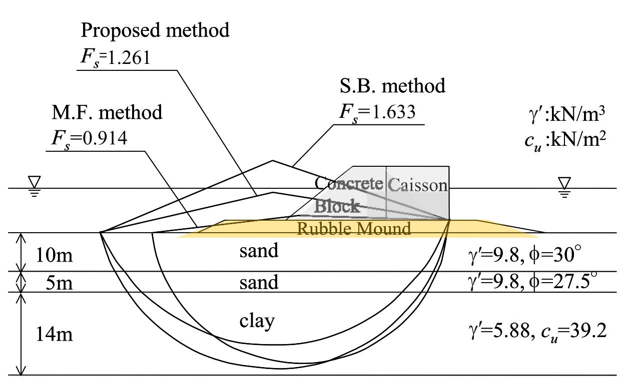

Fig. 17. Cross section and critical slip circles for *Case B* (Kobayashi, 1984).

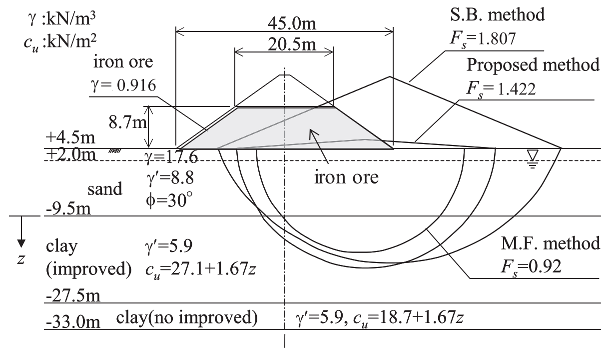

Fig. 18. Cross-section of and critical slip circles for *Case C* (iron ore built in the reclaimed land, Saito (1977)).

Table 4
Comparison of the factor of safety from different methods.

	M.F.	S.B.	CBCF
Case A	0.734	1.590	1.269
Case B	0.914	1.633	1.261
Case C	0.916	1.807	1.422

6. Conclusions

Generally, modified Fellenius' and simplified Bishop's methods based on slip circle of slices have been used for slope stability analyses and calculation of bearing capacity in geotechnical practices over the years and those are found to be popular among the designers and researches elsewhere though there are number of sophisticated methods available in the literature. However, it is well known that the modified Fellenius' method underestimates the factor of safety, while simplified Bishop's method overestimates the factor of safety in the case of ground consists of horizontal sandy layer. In this study, a new slip circle method is proposed for the purpose of calculating the bearing capacity for grounds consisting of sandy layer. Following conclusions are drawn from this study.

(1) A new slip circle method of slices is proposed based on the assumption that the ratio of inter-slice shear force to inter-slice normal force equivalent to $\tan(0.25\alpha_i)$ for all slices. The α_i is the inclination of the slip surface to the direction of resultant forces acting on i^{th} slice. The ratio of inter-slice shear force to inter-slice normal force for simplified Bishop's method is zero while that of modified Fellenius' is $\tan(\alpha_i)$ for slip circle method.
(2) Bearing capacity factors N_c, N_q, and N_γ were calculated for different friction angles from the proposed method. When frictional angle is less than 40°, these values are almost same as that of plasticity solution proposed by Prandtl, which are widely adapted in the technical standards of the infrastructures. The proposed circle slide method is named as CBCF (circle bearing capacity factor) method. When ϕ is greater than 40°, CBCF method becomes to overestimate the bearing capacity factor considerably, and this is the limitation of the method.
(3) CBCF method can be applied to the double layered ground. The results of centrifugal model test, which were carried out for dense sand layer overlying soft clay with various conditions by Okamura et al., were compared with the bearing capacity calculated from CBCF method. The bearing capacity of strip footing and circular footing calculated from CBCF method agreed well with those of model test results within the error of 25%. The failure mode observed in the centrifuge tests of strip footings were closely related the critical sliding surface calculated by CBCF method.
(4) The proposed CBCF method was applied for three case studies for ground consisting of sand and clay layers. It was examined that the factor of safety calculated for the stability of slopes from CBCF method can explain the actual performance of geotechnical structures well.

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